Numerical and experimental analysis of fatigue crack growth under random loading

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Abstract

This article analyses fatigue crack growth under random loading using experimental results taken from literature on the subject and from growth simulations carried out using the Strip Yield Model. The capacity of the Strip Yield Model in representing the retardation effects produced during the growth process is analysed. The effect of different statistical parameters of the random load process and of the representative histories of the same on the crack growth life and on the variability of the results obtained in different representative tests of the same real load process is likewise studied. Some of the parameters considered are the bandwidth of the random loading process, the number of cycles of the load histories employed and the effect of truncating the histories. It can be seen that under certain conditions, the numerical model employed can quite closely predict some of the behaviours of the crack growth. Likewise, it can also be seen that the bandwidth has a significant effect on the fatigue life, and that the length of the histories employed in tests or simulations has a great effect on the variability of the obtained results. It can also be seen that, although the elimination of the overloads tends to give rise to shorter lives, in certain cases, from a statistical point of view, it may be the origin of non-conservative predictions.

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1. Introduction

Fatigue crack growth under random and variable amplitude loading has been analysed by many different authors [1–5]. Some of them pay special attention to the simulation of representative load histories of defined random loading processes [6–8]. Others analyse the effect of overloads and their distribution throughout the load history of the crack growth life [9–11]. There are also many articles that present numerical or analytical models to simulate the crack growth under this type of loading [12–15], or which deal with the effect of many of the other parameters of the loading process on the growth rate [16–20].

Normally, the results of different crack growth tests or simulations under the same random loading process can be different. In the case of tests, this difference in the results is mainly a consequence of the difference in the behaviour of the material employed in each of the specimens and of the randomness of the applied loads. The history applied in each test is different, although they are all representative of the same random loading process. As regards growth simulations, the difference in the results is due solely to the randomness of the loads.

Quite often, the load histories applied in tests or simulations are of a finite duration, and therefore, must be consecutively repeated up to failure. This finite character of these histories is another source of dispersion in the results. Therefore, in order to estimate the crack growth life of any mechanical system it will be important to have the representative load histories of the employed process available and that several tests be executed to enable the expected dispersion in the real results to be estimated. Thus, it will be possible to estimate the real life of the system from a statistical point of view.

One of the objectives of this article is to analyse the effect of some of the statistical parameters of the random loading process on the variability of the results obtained in different crack growth tests under loads that represent the same...
process. The parameters that are taken into consideration are the bandwidth of the random process, the number of cycles in the histories used to represent the random loading process and the statistical distribution of the overloads in the loading history. The results obtained from 130 crack growth tests carried out by the authors on Compact Tension (CT) specimens [21] were employed for this analysis.

Another objective of the article is to analyse the capability of the Strip Yield Model [13] to simulate the fatigue crack growth under irregular and random loading and to reproduce the fatigue life variability produced during the tests. Should the model be capable of reproducing the fatigue life variability, it will be a complement of the tests and will enable the analysis to be extended with a lower test cost.

This article is organised in the following manner. Point 2 provides information on the most significant aspects of the tests carried out. Point 3 presents the results obtained during the tests and includes brief comments on the same. Point 4 analyses the capability of the Strip Yield Model to reproduce the fatigue crack growth under random loading, comparing the simulations to the experimental results. In point 5, the effect of the afore-mentioned parameters on the variability of the results is analysed based on the experimental results and simulations of the fatigue crack growth. Finally, some conclusions are indicated.

2. Test design

The tests were carried out on specimens of 2024-T351 aluminium alloy, employing CT type specimens, with W=50 mm and B=12 mm. All the tests were carried out by making the crack grow from an initial length, \( a_0 = 15 \) mm, to a final one, \( a_f = 25.3 \) mm. The length of the crack was measured continuously using the ACPD technology. Each test was carried out while using a different random loading history, but all of them having the same mean value. Each loading history, \( P_j(t) \), was obtained by adding a constant load \( P_0 = 4850 \) N to a zero-mean random load history, \( P^*(t) \):

\[
P_j(t) = P_0 + P^*(t)
\]

The load histories \( P^*(t) \) are representative of zero-mean stationary gaussian random processes with defined power spectral density functions (psdfs). The value of \( P_0 \) was selected as the minimum needed to avoid compressive loads during any of the tests to be carried out.

In order to obtain \( P^*(t) \), four psdfs have been defined, called AH, BH, CH and DH, each with a different bandwidth. Fig. 1(a) shows the shape of the psdf. The values of the parameters \( \omega_1, \omega_2, \ldots \) and \( h_1, h_2, \ldots \) for the four defined psdfs are shown in Table 1. The area of the four defined psdfs is the same, so that they correspond to random loading processes with the same mean square value. Fig. 1(b) shows samples of \( P^*(t) \) and \( P(t) \), obtained from the psdf AH, together with their probability density functions (pdf). Fig. 1(c) is a plot of the distributions of peaks and troughs of the same loading history \( P(t) \). Finally, Fig. 1(d) shows samples of \( P^*(t) \) and \( P(t) \), obtained from the psdf DH.

The bandwidth of each process has been characterized by means of the irregularity factor \( \varepsilon \), defined as:

\[
\varepsilon = \frac{M_2}{\sqrt{M_0 M_4}}
\]
where $M_n$ is the $n$th order moment of the psdf:

$$M_n = \int_{-\infty}^{\infty} \omega^n S(\omega) d\omega$$  (3)

where $\omega$ is the frequency and $S(\omega)$ the psdf.

The values of $\varepsilon$ corresponding to the four defined psdf, AH, BH, CH and DH are also shown in Table 1.

The mean square of all the zero-mean random processes $P^r(t)$ was $\langle (P^r(t))^2 \rangle = 11,8403 \text{ N}^2$. To summarise, 4 random processes have been defined, $P_{AH}$, $P_{BH}$, $P_{CH}$ and $P_{DH}$, with the same mean square and different bandwidths, with $P_{AH}$ being that with the broadest band and $P_{DH}$ that with the narrowest one. For the specimen geometry and crack lengths considered in the tests, the $P_0$ value produces $K_I$ values ranging from $10.15 \text{ MPa m}^{1/2}$, for the initial crack length, $a_0$, to $17.8 \text{ MPa m}^{1/2}$, for the final crack length, $a_f$.

Of each of the four previously defined random processes 20 different loading histories [22] were simulated, each one with 25,000 cycles. The statistical parameters of each one of the simulated histories for the same loading process are very similar. The maximum difference of the mean value of the peaks or the load ranges between two histories of a same process was less than 0.2%. The standard deviations of the peak values of each history of a process differ in less than 1%. From the $P_{CH}$ random process, two other groups of loading histories were simulated: one with 20 histories of 100,000 cycles each and another with 30 histories of 5000 cycles each. In order to get an idea of the ratio between the maxima of the cycles and their mean value, the values of $K_I$ produced by the average of the maxima have been calculated. As an example, in the case of load histories CH, the average of $K_{I\text{max}}$ ranged from 12.3 MPa m$^{1/2}$, for $a_0$, to 21.6 MPa m$^{1/2}$, for $a_f$.

Fig. 2 shows schematically the process followed for the reconstruction of the loading histories with different bandwidth. It can be resumed as follows: phase 1, four psdfs are generated; phase 2, from each psdf different numbers of random loading histories, $P^{r}_{x}(t)$, with different number of cycles are simulated; phase 3, a constant load, 4850 N, is added to each load history, resulting six different sets of load records. Each history is identified by means of the code XHYFZZ, where X will be A, B, C or D, depending on the bandwidth of the random loading process, YYY represents the number of thousands of cycles of the histories of that group: 5, 25, or 100, and ZZ represents the order number of that history within the group. For example, the history CHF5_12 represents loading history number 12 of the 25,000 cycle history group simulated using the $P_{CH}$ random loading process.

<table>
<thead>
<tr>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Phase 3</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(\omega)$</td>
<td>$P^r_{\text{CH}}$ (25000 cycles long) + 4850 N</td>
<td>20 load hist., $P_{\text{CH}}$</td>
<td>Set DH25</td>
</tr>
<tr>
<td>$S(\omega)$</td>
<td>$P^r_{\text{BH}}$ (5000 cycles) + 4850 N</td>
<td>20 load hist., $P_{\text{BH}}$</td>
<td>Set CH5</td>
</tr>
<tr>
<td>$S(\omega)$</td>
<td>$P^r_{\text{CH}}$ (25000 cycles) + 4850 N</td>
<td>20 load hist., $P_{\text{CH}}$</td>
<td>Set CH25</td>
</tr>
<tr>
<td>$S(\omega)$</td>
<td>$P^r_{\text{AH}}$ (100000 cycles) + 4850 N</td>
<td>20 load hist., $P_{\text{AH}}$</td>
<td>Set AH25</td>
</tr>
</tbody>
</table>

Fig. 2. Schematic process to obtain the load sequences of the histories used.
One hundred and thirty tests were carried out, employing each one of the 130 histories generated previously. Hereinafter, each test will be referred to using the same name as history employed in the same. Further details of the tests and the loading histories generation process can be found elsewhere herein [21].

3. Tests results

Fig. 3(a) shows two sets of $a$–$N$ curves: those produced in the AH25 test group, those with the broadest bandwidth, and in the DH25 group, the process with the narrowest bandwidth. One can see that the dispersion of the results is greater in the AH25 group than in the DH25 group. Fig. 3(b) only indicates the extreme $a$–$N$ curves of the three test groups carried out with histories representative of the CH process: CH5, CH25 and CH100. In the same, one can see that the dispersion increases when the number of cycles of the loading histories is reduced.

Fig. 4(a) shows the distribution of the lives obtained in all the tests carried out in each of the four groups with loading histories with the same number of cycles: AH25 to DH25, on lognormal probabilistic paper. One can see that there is a good adjustment to this type of distribution. In order to analyse better, the effect of the number of cycles of the histories, the distributions of the crack growth lives obtained in the CH5, CH25 and CH100 tests, $N_5$, $N_{25}$ and $N_{100}$, respectively, are shown in Fig. 4(b).

As has been indicated, the variability of the results depends mainly on the difference in the behaviour of the material employed in one specimen and another, and on the randomness of the loading histories. In order to distinguish which part of the variability depends on one parameter and which part depends on another, the authors [21] carried out two new series of tests: one applying one loading history from the CH5 series in all the tests of the series, called CH5* and another one applying one loading history from the CH25 series in all the tests of the series, called CH25*. Each test of these two series, CH5* and CH25*, resulted in a different life, $N_5^*$ and $N_{25}^*$, respectively. As the same history of loads was used in all tests in a group, it was assumed that the randomness in the material was the sole independent variable influencing the dispersion of life. On the other
Table 2 shows the variances of log $N$, log $N'$ and log $N_L$ [21].

<table>
<thead>
<tr>
<th></th>
<th>5000 Cycles</th>
<th>25,000 Cycles</th>
<th>100,000 Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Var} \log N$</td>
<td>$1.39 \times 10^{-3}$</td>
<td>$1.78 \times 10^{-4}$</td>
<td>$9.04 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\text{Var} \log N'$</td>
<td>$9.62 \times 10^{-5}$</td>
<td>$13.8 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>$\text{Var} \log N_L$</td>
<td>$1.3 \times 10^{-3}$</td>
<td>$4.0 \times 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>

hand, the randomness of the material and of the load histories was assumed to be the two independent variables influencing the dispersion of life for the tests series CH5, CH25 and CH100. From these results, the authors estimated the variance of the life inherited from loads in the tests series CH5, CH25. Table 2 shows the variances of log $N$ and of log $N'$ obtained in tests, as well as the estimated variance of the life inherited from loads, $\text{Var} \log N_L$ [21]. It can be seen that the dispersion obtained in the CH100 series is similar to that produced by the variability of the material and thus the part of the variability of the results due to the material cannot be distinguished from that due to the load histories. They also verified the fact that in the CH25 series the variability of the results is in the order of twice that produced by the material, while in the CH5 series the contribution of the variability of the material to the total variability can be neglected.

4. Simulations with the Strip Yield Model

The use of analytical or numerical models to simulate crack growth under random loading is very important to reduce the number of tests required for any fatigue crack growth analysis. One of the requirements that any model must meet is to be capable of simulating the crack growth process with sufficient precision and safety, estimating the effects of the overloads and their sequence.

In order to complement the experimental analysis presented in this paper, many simulations by using the Strip Yield Model [13] have been carried out. This model is based on the analytical approach proposed by Budiansky and Hatchinson [23], using the Dugdale approach to calculate the crack tip plastic zone. To be able to distinguish between plane stress and plane strain conditions when calculating the plastic zone size, the model uses the constraint factor $\alpha$, for tension, and $\beta$, for compression. These constraint factors range from 1 for plane stress conditions to 3 for plane strain. The model is implemented in the FASTRAN II code [24]. This code allows to define any value for $\alpha$ and $\beta$. They must be defined, depending on the stress conditions expected, which will depend on the thickness of the specimen and the stress level. In case of variable amplitude loading, the code can also define automatically, cycle by cycle, a variable constraint factor $\alpha$, which will be defined as a function of the maximum value of the stress intensity factor in the cycle. For the material and thickness used in this paper, $\alpha$ ranged from 1.73 for

$K_{I,max} \leq 8.7 \text{ MPa m}^{1/2}$, to 1.3 for $K_{I,max} \geq 16.9 \text{ MPa m}^{1/2}$, with $\alpha$ varying linearly between them.

Prior to employing the code to extend the experimental analysis, its capacity has been verified simulating the behaviour of the cracks during the course of the 130 tests that have been made.

Beforehand, the authors analysed [25] the effect of the constraint parameters $\alpha$ and $\beta$ on the quality of the results obtained by simulation. They came to the conclusion that the best approximation to the mean of the obtained lives in any set of tests was produced by employing a variable constraint factor, $\alpha$. However, the variability of the results of the simulations in each group, which is due only to the randomness of the loads was much greater than that produced experimentally, which is a consequence of the randomness of the load and the material. By using different constant values of the $\alpha$ and $\beta$ parameters, the authors verified the fact that shorter lives were produced. However, the approximations to the variability of the results in each group of tests were better. They came to the conclusion that the best approximations to the variability were obtained with $\alpha=1.5$ and $\beta=1$. Although, the analysis was made with different length histories, the results are mainly based on the results obtained with 5000 cycle histories. One must take into account the fact that the shorter the length of the history the importance of the effect of the randomness of the material in relation to that produced by the randomness of the load [21] is reduced (see Table 2). Table 3 provides some of the results obtained during the aforementioned analysis.

Fig. 5 shows the experimental results of each one of the CH5 tests, compared with three different simulations: one with $\alpha$ variable, another with $\alpha=1.2$ and $\beta=1$ and another with $\alpha=1.5$ and $\beta=1$. In the same, one can see how $\alpha$ variable produces better approximations of the life, but worse ones of the variability of the life in each test group. It can be seen that with $\alpha=1.5$ and $\beta=1$, the evolution of

<table>
<thead>
<tr>
<th>Constraint parameters</th>
<th>Mean life, $\mu_N$ (cycles)</th>
<th>Standard deviation, $\sigma_N$ (cycles)</th>
<th>estimated $\mu_N$/experimental $\mu_N$</th>
<th>estimated $\sigma_N$/experimental $\sigma_N$</th>
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<tr>
<td>$\beta$</td>
<td>$\alpha$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.2</td>
<td>116,508</td>
<td>24,466</td>
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</tr>
<tr>
<td>1.3</td>
<td>1.7</td>
<td>107,254</td>
<td>22,993</td>
<td>0.63</td>
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<tr>
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<td>1.7</td>
<td>91,644</td>
<td>15,459</td>
<td>0.54</td>
</tr>
<tr>
<td>1.7</td>
<td>2.1</td>
<td>83,202</td>
<td>6107</td>
<td>0.49</td>
</tr>
<tr>
<td>Variable</td>
<td></td>
<td>84,017</td>
<td>2538</td>
<td>0.49</td>
</tr>
<tr>
<td>1.3</td>
<td>1.7</td>
<td>145,332</td>
<td>41,231</td>
<td>0.86</td>
</tr>
<tr>
<td>1.5</td>
<td>1.7</td>
<td>117,253</td>
<td>29,540</td>
<td>0.69</td>
</tr>
<tr>
<td>Tests</td>
<td>169,978</td>
<td>15,272</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the simulations is similar to that of the tests, but displaced 78,000 cycles, approximately.

In order to see the model’s capability to reproduce the effect of the overloads and underloads on the growth rate, it is interesting to compare the growth rate during the tests and simulations. The achieving of a good approximation in the retardation and acceleration effects is fundamental to approximate the variability of the life in the tests with histories of the same statistical characteristics. Fig. 6 shows the evolution of the crack growth rate produced in four different tests, one from each group of tests carried out with histories of 25,000 cycles. Fig. 7 shows a comparison between the evolution of the crack growth rate (\(da/dN\)) in a test from the CH25 series, curve a, and those produced in the simulation of the same test with \(\alpha\) variable and \(\alpha = 1.5\), curves b and c, respectively.

It can be seen that the crack growth rates are different, although the effects of the overloads are reproduced to a certain extent. It can also be seen that there is a pattern in the crack growth rate, which is repeated and amplified when the crack grows. It is repeated as a consequence of the repetition of the load sequence every 25,000 cycles. Although, the repetition of the pattern can be seen in the three curves, it is clearer in tests than in simulation. One of the reasons for that behaviour is because the crack growth rate is higher in simulation, thus the crack length increase after 25,000 cycles is larger in simulation than in test. But, if the crack length is different from one repetition to the next, the effect of the load sequence will be different, and the difference will be larger as the crack length ratio increases from one repetition to the next.

In order to enable an easier comparison, each one of the simulated growth rate curves can be multiplied by the ratio between the tendency of the growth rate in the test and that of the corresponding simulation. To do that, a polynomial has been fitted to each growth rate curve. Later on, a new curve with the ratio of these fitted curves (test/simulation) has been calculated. Finally, the actual crack growth rate curve obtained by simulation has been multiplied by the new curve representing the above mentioned ratio. This will result in the curves being almost superimposed, which will enable an easier comparison of the produced retardations and accelerations. Fig. 8(a) shows the test \(da/dN\) curve and that of the simulation with \(\alpha = 1.5\) subsequent to the indicated transformation, and Fig. 8(b) shows that of the test and that of the simulation with \(\alpha\) variable. Fig. 9 represents a detail between 25,000 and 50,000 cycles where the simulated \(da/dN\) have been adjusted to obtain the same mean rate during these 25,000 cycles, to facilitate the comparison.

One can see, in Figs. 7–9, that the simulation with \(\alpha\) variable overestimates the effect of the overloads, producing very high retardations. However, subsequent to the retardation period, the growth rate becomes greater than in the tests. In the case of \(\alpha = 1.5\), the retardation effect is much less accused than with \(\alpha\) variable, producing retardations that are more similar to the experimental ones and growth rates subsequent to the retardation that are slightly greater.
than the experimental ones. One must take into account the fact that these effects that have been described are produced in simulation with higher average growth rates.

From these results one can conclude that, in general, the simulation with a variable produce better approximations of the mean life of all the tests of a group than the simulations with a constant. However, a variable constraint factor, $\alpha$, does not produce good approximations to the variability of the life in a group of tests. To the contrary, a constant constraint factor, $\alpha = 1.5$, produces bad approximations of the mean life of the test group, but a good approximation of the variability. Some considerations can be made as to the causes why the simulation with $\alpha$ constant underestimates the fatigue lives under the conditions of the simulated tests. The authors are currently working on a modification of the model, including other types of closure, that improve the life predictions, maintaining the quality of the prediction of the variability of the results.

Likewise, it must be said that, in the cases studied, the Strip Yield Model is capable of predicting with a certain approximation the effects produced by the overloads and their sequence within a random loading history. The approximation achieved can be considered as being sufficient to decide to use the model in the study of the influence of certain parameters on the fatigue life.

In accordance with the above analysis, hereinafter, all the complementary simulations to the tests to study the variability of the results obtained with different histories that are representative of a same process shall be done with $\alpha = 1.5$ and $\beta = 1$. 

5. Analysis of results

One can see in Fig. 4(b) that the mean life obtained in the three test groups, carried out with the same random loading process, is very similar. However, in this figure as well as in Fig. 3(b), one can see the great influence of the number of cycles of the histories employed to represent the loading process on the variability of the results. One can see that the variability of the lives obtained in the CH5 group is quite a bit greater than in the CH25 group, and this, at the same time, is greater than the CH100 group. When analysed from a statistical point of view, this implies that for low probabilities of failure, the number of cycles for the failure in the CH5 group will be quite a bit lower than in the CH100.
group, although the two groups represent the same random loading process. One can see, in Fig. 4(b) that, for example, the number of cycles at which the 1% failure will be produced will be 133,814 in the case of using 5000 cycle histories and 159,495 should 100,000 cycle histories be used.

As has already been indicated, loading histories with a different number of cycles were employed for each one of the three test groups indicated. However, as has likewise been indicated, the maximum difference in the mean and the standard deviation of peaks between two histories of a same random process was negligible. Therefore, one can verify that in order to obtain dispersions similar to the real ones, or to those produced with histories with a sufficiently high number of cycles, it is not sufficient to use histories proceeding from the real random process and guarantee that the mean and standard deviation of peaks are practically the same as the real ones. Moreover, the similitude of other statistical parameters of the histories will need to be guaranteed.

Considering the great effect that the overloads have on the crack growth, it will be important to analyse the tails of the load histograms of the histories employed. The variability of the tails of the histograms from one history to another will be indicative of the variability of the obtained live. In real cases, the variability of the life under the same random loading process will be very much influenced by the variability of the tails of the applied load histograms, which at the same time will be a function of the number of life cycles of the specimen under analysis. The variability of these tails can be determined by simulating length histories equal to the real life. In tests simulating real loads it must be expected to have a crack growth life variability close to the real one if the tail of the distribution of peaks possess a variability similar to the real one.

Normally, in order that an overload produce an appreciable retardation effect, its value must be sufficiently high in comparison to the loads applied thereafter. Taking this into account, it has been estimated that the load peaks with a value greater than 1.5 times the mean of the peaks of the random process to which the history belongs, $\mu_1 (> 1.5 \mu_1)$, will be important due to their retardation effect. Fig. 10(a) shows the tail of the probability density function of peaks of the random process CH, $p_{CH}(x)$, for peak values greater than 1.5 $\mu_1$. Likewise, it shows the tails of the histograms of the peaks of three load histories, one from each test group: CH5, CH25 and CH100 ($p_{CH5_5}(x)$, $p_{CH25_11}(x)$, $p_{CH100_11}(x)$). The vector $\mathbf{x}$ represents the values of the centres of the intervals into which the peaks have been classified. In the figure, one can see the enormous difference between the tail of the probability density function of peaks and that of the histogram corresponding to the CH5 history. The differences between the histograms of this group are similar to those produced as regards the tail of the probability density function. One can also see that the tails of the histograms of the CH25 and CH100 groups are quite a bit more similar to the one corresponding to the probability density function. As occurs with the CH5 histories, the difference between the histograms of each one of these two groups is in the same order as those produced as regards the probability density function one.

In order to characterise the variability of the tails of the peaks histogram of each loading history, the parameter $y_{ij}$ has been defined, which can be expressed as (Fig. 10(b)):

$$y_{ij} = \frac{1}{n} \sum_{k=1}^{n} (p_{ij}(x_k) - p(x_k))^2$$

where $i$ represents the test group to which the history belongs (AH25, CH5, CH25…), $j$ represents the number of the history within the group, $n$ the number of intervals in which the range of the variation of the values of the peaks has been divided and $x_k$ the central value of each one of the intervals. The greater the value of the $y_{ij}$ parameter, the greater the dispersion of the number of peaks of the history will be in relation to the values of the probability density function, $p(x)$.

![Fig. 10. (a) Tails of the CH process probability density function and the histograms of three histories of the CH5, CH25 and CH100 groups; (b) data to define the $y_{ij}$ parameter.](image-url)
The variability between the tails of all the histograms of a test group, \( i \), can be characterised by the mean of the \( y_{ij} \) parameter of each group of histories:

\[
\mu_{yi} = \frac{\sum_{j=1}^{M_i} y_{ij}}{M_i}
\]  

(5)

where \( M_i \) is the number of load histories of each group.

Likewise, the variability of the lives in each test group, \( i \), can be characterised by means of the standard deviation, \( \sigma_{Ni} \), of the \( N_{ij} \) lives obtained in each test group, \( i \). Fig. 11 shows a graph of \( \sigma_{Ni} \) versus \( \mu_{yi} \) for the three CH test groups. The close relationship between these parameters can be seen therein.

In order to isolate the randomness of the loads effect, the tests can be simulated using the Strip Yield Model. If the model is capable of simulating the effects of the overloads and underloads and their sequence, the results will quite correctly represent the variability of the life, but without including the effect of the randomness of the material, which is not included in the model. Fig. 11 also shows the relationship between the standard deviation of the lives simulated, \( \sigma_{Ni} \), in the three groups and \( \mu_{yi} \). It is easy to see that in the case of the simulations, where the standard deviation does not depend on the randomness of the material, the variability of the lives in the CH25 and CH100 groups is clearly lower than in the experimental case. Moreover, one can see that the dependence of \( \sigma_{Ni} \) with respect to \( \mu_{yi} \) is greater.

From the results of the tests and simulations of the CH5 group, we have been able to verify the fact that the extreme value of the loads in each history has a great influence on the life produced in the corresponding test. However, this influence is negligible when the length of the history is sufficiently long. Fig. 12 shows the correlation between the life and the extreme value of the load in the history employed for the CH5 and CH100 test groups. One can see therein the large correlation that exists for the group of short histories and how small it is in the case of those with long ones.

Fig. 13(a) shows the evolution of the growth rate obtained by simulation of two CH5 tests: that which produces the longest life and that which produces the shortest one. Fig. 13(b) shows a detail of the above-mentioned, where one can better appreciate the behaviour. One can see that not very many peaks produce a significant retardation effect, which has significant consequences on the crack growth. One can also see that in the case of longer fatigue life, the greatest overload has such a retardation effect that it is practically maintained until it is repeated again some 5000 cycles further on. In the case of the shortest simulation, two overloads produce the effect instead of one, although much less. This behaviour explains the close correlation between the extreme value of the history and the life obtained with that history.

If the number of cycles of the load history is so short that the effect of the extreme value is maintained practically throughout the whole history, its effect will be enormous. In that case, the extreme value has a great effect on the produced life, as can be seen in Fig. 13. To the contrary, if the history is long enough for there to be sufficient cycles that produce typical effects of the overloads, then it will be that set of overloads which will have a certain effect on the life. In this case, the retardation effect will be produced by the set of highest overloads of each history. This effect can be seen in Figs. 6–9. While the number of cycles that generate a significant sequence effect increases, the mean and the standard deviation of that group of peaks in a history will be more similar to those of the rest of the histories. This means that the variability of the lives obtained in the different tests of the group is much lower.
The fact that greater variability is produced in the results with the lower number of cycles produces, in any case, results that are safer, given that the design is made considering the lower tail of the life distribution, which is lower for the short histories. However, it can be over-conservative.

Another parameter that greatly influences the variability of the results obtained in each test group is the bandwidth of the loading process employed. Fig. 4(a) shows the distribution of the lives obtained in the four test groups with 25,000 cycle histories on lognormal probabilistic paper. Fig. 14 shows the COV \((\text{COV} = \sigma_{N_i}/\mu_{N_i})\) obtained in the four test groups versus \(\varepsilon\). One can see in these two figures that upon the bandwidth increasing (reducing \(\varepsilon\)) the mean life of the specimens and the variability of the results increase. Nonetheless, the influence of the bandwidth on the variability of the results is not excessive. The increase of the mean life, \(\mu_{N_i}\), with the bandwidth is a consequence of the fact that, for random processes with the same mean square, when the bandwidth increases a reduction in the mean value of the peaks, \(\mu_p\), and of the load ranges is produced. The increase of the variability of the results, \(\sigma_{N_i}\), with the bandwidth is partly related to the effect of the overloads and their number in each load history.

In order to see the influence of the load peaks greater than 1.5 \(\mu_p\), three series of simulations of the four test groups with 25,000 cycles under three different conditions have been carried out. The first series used the histories employed in the tests. The second and third series of simulations have been carried out modifying the peaks that have values greater than 1.5 \(\mu_p\). For the second series of simulations the highest peaks of all the histories have been modified. In each group of tests the tails of the histograms of peaks \((x > 1.5 \mu_p)\) of all the histories have had to be identical and equal to the probability density function, \(p(x)\), of the random process represented by the group of histories. Therefore, the effect of the values of the peaks in those tails on the variability of the results has been practically eliminated. The third group of simulations has been carried out after eliminating all peaks above the value 1.5 \(\mu_p\).

Fig. 14 also shows the COVs obtained in the three series of simulations with the four groups of histories versus \(\varepsilon\). Fig. 15 shows a comparison of the lives obtained in each one of the simulations of the three series. Fig. 16 shows the same results on lognormal probabilistic paper, where one can better appreciate the life distributions obtained. One can see that the variability is noticeably reduced when the tails are adjusted to \(p(x)\) (the standard deviation is divided approximately by 2), which demonstrates the large effect the tails have on the dispersion. On the other hand, one can see that the effect of the tails of the distributions of peaks in the dispersions is similar for all bandwidths, given that the reduction is of the same order in all the groups. Likewise, this is to say that by employing the original histories the dispersions will be clearly reduced if the number of cycles of the histories is increased, given that in this case the distributions of the tails will be more similar among themselves and to the tail of the rated \(psdf\) of the random
Fig. 15. Crack growth lives obtained in the three series of simulations of the AH25, BH25, CH25 and DH25 test groups.

Fig. 16. Distributions of the crack growth lives obtained in the three series of simulations of the AH25, BH25, CH25 and DH25 test groups, represented on lognormal probabilistic paper.

load process being dealt with. In any event, one must remember that the increase in length of the histories always has a limit, which is the life of the specimen.

If one analyses the variability of the groups of simulations carried out after eliminating the peaks with values above 1.5 \( \mu_x \), one can see that the dispersion tendency is modified with the bandwidth (Fig. 14). This means to say that the tails, even with the same distribution in all the histories of any one group, continue to have an effect that increases the variability with the bandwidth. This is an effect that could be due to the sequence of the overloads, given that, although their value was modified, they were kept in the position they had in the original histories. Therefore, the position of the high peaks was not the same in all the modified histories. Nonetheless, it can be seen that the dispersion in the cases of the modified histories is much less than that produced by the randomness of the material employed in the tests.

Finally, one must stress that eliminating the peaks with values above 1.5 \( \mu_x \) from the histories produces, in all the simulations, lives that are equal to, or shorter than those obtained with the original history. However, if one considers the set of simulations of any one group, and therefore, of only the one random process, one can verify the fact that the truncating of a load history does not guarantee the fact that shorter lives than those in any of the real histories are going to be produced. This fact can be more clearly seen in Fig. 16. One can see therein, especially as regards narrow bandwidth, that the probabilities that shorter lives be produced with the original histories than those obtained with truncated histories can be noticeable.

6. Conclusions

The results of 130 fatigue crack growth tests under random loading employing histories that are representative of several random processes have been analysed. The results of the tests have been compared with simulations carried out by means of the Strip Yield Model, and many additional simulations have been made to complement the results of the analysis. Some conclusions can be drawn from all these results and these analyses.

As regards the usefulness of the Strip Yield Model, it may be said that:

1. The comparison of the variations of the crack growth rate during the growth under irregular loads allows one to say that this model qualitatively reproduces the effects produced by the overloads within the histories.
2. By employing a variable constraint factor, \( \alpha \), the mean lives for a test group are quite approximate to the real ones. However, the model overestimates the variability of the results within the group.
3. On the other hand, by using a determined constant constraint factor value, the model is capable of making a close estimation of the variability of the lives obtained in a set of tests. However, the mean life in the set is underestimated.
4. The Strip Yield Model can be a suitable tool for analysing the effect of different parameters on the distribution of the fatigue life under random loads. It might also be useful to define the load histories that are representative of a random loading process. A history that may be used with to predict the fatigue behaviour of elements in real situations.
5. But, in general, it can be said that the predictions of the Strip Yield Model for crack growth life are not so good to consider the model satisfactory. It would be interesting to analyse the possibility of including the effect of other kind of closure, such as oxide or roughness induced closure, in the model to improve the quality of the predictions.

As regards the effect of different statistical parameters of the histories employed to represent the random loading processes on the crack growth behaviour, it can be said that:

6. For one defined random loading process, the use of very short histories produces approximately the same mean value of the lives obtained in a test group as when longer histories are employed. However, it produces greater dispersions than the longer histories, which are more similar to reality. In the event of designing to obtain very low probabilities of failure, this increase of the dispersion will lead to conservative results.
7. For one defined random loading process, the dispersion of lives produced in different tests decreases when the length of the load histories increases. Considering only the peaks with a value greater than 1.5 \( \mu_x \), the mean of the \( y_{i,j} \) parameter, \( \mu_{y_i} \), is strongly correlated to the dispersion of the lives obtained.
8. The bandwidth of the random loading process has a great influence on the lives obtained in a group. Likewise, in this case, \( \mu_{y_i} \) is strongly correlated to the dispersion.
9. The truncation of the load histories, eliminating the highest peaks, reduces the life obtained in the tests. However, in spite of this reduction, the probability of having real lives lower than those obtained with truncated load histories may still be significant.

References


