Abstract—This paper presents a multi-period probabilistic production cost model. Dispatch intra-period and inter-period constraints are considered through the use of a facet LP formulation. An efficient solution procedure based on the Dantzig–Wolfe decomposition technique is developed. The work reported in this paper extends previous work in two respects: (i) inter-period constraints are rigorously treated, and (ii) a systematic general solution procedure is developed. Results for a large-scale case study are presented.

Index Terms—Multi-period probabilistic production costing, dispatch constraints, facet LP, Dantzig–Wolfe decomposition.

NOMENCLATURE

The notation used is stated below.

Indices:
i plant index,
j intra-period constraint index,
m inter-period constraint index,
k time period index,
s index for the initial solutions,
l iteration index,
ν current iteration index,
I + 1 index of the fictitious unserved-energy plant.

Sets:
Ω any set of plants,
Ω0 set of initial solutions of the subproblem associated to period k,
Ξνk set of positive reduced cost solutions of the subproblem associated to period k from iteration 1 to iteration ν − 1,
∅ the empty set.

Numbers:
I number of plants,
Jk number of intra-period constraints in period k,
M number of inter-period constraints,
K number of periods.

Variables:
ei,k expected energy produced by plant i in period k,
ψλk weighting factor for the solution of the subproblem associated to period k at iteration λ,
ei,j,l expected energy produced by plant i in the subproblem associated to period k at iteration λ,
ν objective function of the original problem,
ν objective function of the master problem at iteration ν,
νk objective function of the subproblem associated to period k at iteration ν.

Marginal values:
δλk reduced costs of the subproblem of period k at iteration λ,
λm,ν marginal value of the inter-period constraint m at iteration ν,
ρj,ν marginal value of the intra-period constraint j of period k at iteration ν,
cj,ν marginal value of the convexity constraint for period k at iteration ν.

Constants:
Ci running cost of plant i,
Dm,j,ki coefficient of variable ei,k in the inter-period constraint m,
Ai,j,ki coefficient of variable ei,k in the intra-period constraint j of period k,
Fm right-hand side of the inter-period constraint m,
Dj right-hand side of the intra-period constraint j of period k,
Wk(Ω) expected unserved energy after loading plants of set Ω in period k,
PKl expected cost of the subproblem associated to period k at iteration l,
Qm,li contribution to the right-hand side of the inter-period constraint m of solution l of the subproblem associated to period k,
Rj,ki,l contribution to the right-hand side of the intra-period constraint j of solution l of the subproblem associated to period k.

1. INTRODUCTION

PROBABILISTIC production cost models [1]–[4] are widely used tools to estimate expected electric energy productions and expected costs in a large/medium-term horizon. A typical time horizon is a year divided in monthly or weekly periods.

The load in every period is modeled using a load duration curve. Load duration curve models offer reasonable accuracy
for moderate effort. The load duration curve expresses the probability that the load is expected to equal or exceed a fixed power value.

Plants are ordered from lower to higher operating cost. This order is often referred to as merit order. This merit or loading order is used to load plants at maximum capacity. This plant loading rule is enough to carry out a production cost simulation so that total production costs are minimized. However, this rule holds only if no dispatch constraints are taken into account.

Plant outages are taken into account by incorporating into the load duration curve the additional demand imposed by outages of plants with lower positions in the loading order. The equivalent load duration curve obtained in this way expresses the probability that the equivalent load (customer load plus outage loads of lower loaded plants) is expected to equal or exceed a fixed power value.

The cumulant technique [5], [6] or the mixture of normals approximation technique [7] are typically used to efficiently implement an equivalent load probabilistic production cost model. In the work reported in this paper a cumulant technique is used.

The equivalent load framework makes it difficult to model dispatch constraints. Intra-period constraints, i.e., constraints that couple together decisions within a given period, are typically modeled using heuristic rules [4], [9]. Intra-period dispatch constraints include the ones involved in: (i) energy storage plants, (ii) must-run requirements, (iii) multiple block plants, (iv) two area systems with a transmission limit, (v) limited energy plants, and (vi) dispatchable plants. The dispatch constraints above are the reason why Bloom & Gallant [8] proposed a framework, that retaining all the advantages of the equivalent load technique, makes it possible to model intra-period linear dispatch constraints.

In the models reported in the literature, time periods are considered independent, and therefore inter-period constraints, i.e. constraints which couple together decisions involving several periods, are not considered. Multiple period dispatch constraints include the ones involved in: (i) emission caps over multiple plants, and (ii) complex cascaded hydroelectric plants. Inter-period constraints are particularly useful to model dispatch restrictions of the cascaded hydroelectric power plants of a complex river system.

This paper addresses a multi-period probabilistic production cost model including both intra-period and inter-period constraints. These constraints are rigorously treated and an efficient solution procedure is developed to solve the multi-period probabilistic production cost problem.

The work reported in this paper is built upon and extends and generalizes the facet LP formulation of Bloom & Gallant [8].

Previous reported work is extended in two respects: (i) the traditional single-period probabilistic simulation production cost model is extended to a multi-period frame-work, and (ii) an efficient solution procedure, based on the Dantzig-Wolfe decomposition technique, is developed to solve the multi-period problem.

This paper is organized as follows. Section II formulates the dispatch constrained multi-period probabilistic production cost model addressed in this paper as a facet LP problem including additional linear constraints. In Section III the proposed solution technique is developed. In Section IV results for an example and a realistic large-scale case study are reported. Section V provides conclusions.

II. Problem Formulation

The multi-period probabilistic production cost model including dispatch constraints is formulated below.

\[
\text{Minimize} \sum_{i=1}^{K} \sum_{k=1}^{I+1} C_i^k e_i^k = z \quad (1)
\]

subject to

\[
\sum_{k=1}^{I+1} D_i^m e_i^k = F^m; \quad m = 1, \ldots, M \quad (2)
\]

\[
\sum_{j=1}^{K} A_{i,j} e_i^k = D_j^k; \quad j = 1, \ldots, J; \quad k = 1, \ldots, K \quad (3)
\]

\[
\sum_{i \in \Omega} e_i^k \leq W^k(\emptyset) - W^k(\Omega); \quad \forall \Omega \subset \{1, \ldots, I + 1\}; \quad k = 1, \ldots, K \quad (4)
\]

\[
\sum_{i=1}^{I+1} e_i^k = W^k(\emptyset); \quad k = 1, \ldots, K \quad (5)
\]

\[
e_i^k \geq 0; \quad i = 1, \ldots, I + 1; \quad k = 1, \ldots, K \quad (6)
\]

Decision variables are the expected energies produced by every plant in every period.

Equation (1), the objective function, represents the addition of expected generation costs over plants and periods.

Block of equations (2) represents inter-period constraints, i.e. constraints that couple together the production of a set of plants over different time periods. These constraints are particularly useful to model hydro system constraints. They allow the optimal allocation of hydro generation among periods.

Block of equations (3) represents intra-period constraints, i.e. constraints that couple together the production of a set of plants within a given time period. These constraints are used to model environmental and different types of dispatch constraints.

Block of equations (4) and equation (5) are the facet constraints [8] used to express a probabilistic production costing model as a linear programming problem.

Finally, block of constraints (6) enforce the positiveness of energy values.

It should be noted that \(W^k(\Omega)\) is the expected unserved energy value of period \(k\) after loading the plants in set \(\Omega\). To compute this energy value a conventional probabilistic simulation run [1] has to be performed.

This problem cannot be solved directly. The number of constraints grows exponentially with the number of plants and, for systems of realistic sizes, it reaches extremely high values. A 12 period case study including 100 generating plants requires \(12 \times (2^{100+2} - 1) = 3.04 \times 10^{31}\) facet constraints. Therefore,
III. SOLUTION APPROACH

Blocks of constraints (2) and (3) are the complicating constraints. They make the problem formulated in the previous section hard to solve. If these blocks of constraints are ignored, the resulting problem decomposes by time period, and every subproblem attains such a structure that it can be solved in a straightforward manner by direct application of the Balériaux technique [1]. However, blocks of constraints (2) and (3) prevent the direct use of the Balériaux algorithm.

The Dantzig–Wolfe decomposition technique was developed to efficiently solve problems with the structure of problem (1)–(6). This technique guarantees the optimality of the solution found [10], [11]. Through the Dantzig–Wolfe decomposition procedure the original problem is reformulated becoming the so-called master problem. In this master problem complicating constraints are explicitly considered, while the remaining constraints are implicitly considered. The master problem typically has a low number of constraints but a high number of variables; this is why it is solved using a column generation strategy.

The variables (columns) to add to the master problem at every iteration are determined through the solution of the subproblems. Every subproblem is associated to a time period and includes only noncomplicating constraints. The solution of every subproblem is, therefore, independently obtained by straightforward application of the Balériaux technique [1].

A. The Master Problem

The master problem at iteration \( \nu \) has the form:

\[
\text{Minimize}_{\nu_{s}^{k}, \mu_{s}^{k}, \lambda \forall s, \lambda \forall k} \sum_{k=1}^{K} \left( \sum_{s \in \psi_{0}^{k}} P_{s}^{k} \nu_{s}^{k} + \sum_{k \in \Xi_{k}} P_{k}^{k} \mu_{k}^{k} \right) = z_{\nu} \quad (7)
\]

subject to

\[
\sum_{k=1}^{K} \left( \sum_{s \in \psi_{0}^{k}} Q_{m,s}^{k} \nu_{s}^{k} + \sum_{k \in \Xi_{k}} Q_{m,k}^{k} \mu_{k}^{k} \right) = F_{m} \cdot \lambda_{m, \nu}; \quad m = 1, \ldots, M \quad (8)
\]

\[
\sum_{s \in \psi_{0}^{k}} R_{s,s}^{k} \nu_{s}^{k} + \sum_{k \in \Xi_{k}} R_{k,k}^{k} \mu_{k}^{k} = D_{j}^{k} : \mu_{j, \nu}; \quad j = 1, \ldots, J^{k}, k = 1, \ldots, K \quad (9)
\]

\[
\sum_{s \in \psi_{0}^{k}} \nu_{s}^{k} + \sum_{k \in \Xi_{k}} \mu_{k}^{k} = 1 : \sigma_{k}^{k}; \quad k = 1, \ldots, K \quad (10)
\]

\[
\nu_{l}^{k} \geq 0 ; l = 1, \ldots, \nu - 1; k = 1, \ldots, K \quad (11)
\]

where

\[
\Xi_{k} = \{i \in \{1, 2, \ldots, \nu-1\} : d_{i}^{k} > 0\}; k = 1, \ldots, K \quad (13)
\]

is the set of positive reduced cost solutions of the subproblem associated to period \( k \) from iteration \( 1 \) to iteration \( \nu - 1 \),

\[
d_{i}^{k} = \sigma_{i}^{k} - \zeta_{i}^{k} \quad (14)
\]

are reduced costs,

\[
P_{k}^{k} = \sum_{i=1}^{I+1} C_{i} \chi_{i}^{k} \quad (15)
\]

is the contribution to the total cost of solution \( I \) of the subproblem associated to period \( k \),

\[
Q_{m,i}^{k} = \sum_{i=1}^{I+1} D_{m,i}^{k} \chi_{i}^{k} \quad (16)
\]

is the contribution to the right-hand side of the inter-period constraint \( m \) of solution \( I \) of the subproblem associated to period \( k \),

\[
R_{i,j}^{k} = \sum_{i=1}^{I+1} A_{i,j}^{k} \chi_{i}^{k} \quad (17)
\]

is the contribution to the right-hand side of the intra-period constraint \( j \) of solution \( I \) of the subproblem associated to period \( k \).

The variables of the master problem are the weighting factors for the solutions of the subproblems. The objective function (7) is a convex combination of the available subproblem solutions. Equations (8) enforce inter-period constraints while equations (9) enforce intra-period constraints. Equations (10)–(12) enforce convex combination conditions for every period.

It should be noted that the master problem above is a small size LP problem whose constraint number is constant and equal to the number of complicating constraints plus one convexity constraint for each period, and whose variable number grows with the number of iterations. A revised simplex strategy can be implemented to keep the member of variables constant and equal to the number of constraints. However, this strategy significantly deteriorates the computational efficiency of the whole procedure because the number of Dantzig–Wolfe iterations increases.

B. The Subproblems

The subproblem at iteration \( \nu \), associated to period \( k \) has the form:

\[
\text{Minimize}_{\nu_{s}^{k}, \mu_{s}^{k}, \forall i} \sum_{i=1}^{I+1} \left( C_{i} - \sum_{m=1}^{M} \lambda_{m, \nu} D_{m,i}^{k} - \sum_{j=1}^{J^{k}} \mu_{j, \nu} A_{i,j}^{k} \right) \chi_{i}^{k} \quad (18)
\]

subject to

\[
\sum_{i \in \Omega} \chi_{i}^{k} \leq W_{k}(\Omega) - W_{k}(\Omega); \quad \forall \Omega \subset \{1, \ldots, I+1\} \quad (19)
\]
\[ \sum_{i=1}^{I+1} c^k_{i, \nu} = W^k(\emptyset) \quad (20) \]

\[ c^k_{i, \nu} \geq 0; \quad i = 1, \ldots, I + 1 \quad (21) \]

The variables of the subproblems are the expected energies produced by every plant. The cost associated to every variable in the objective function consists of three terms: (i) running cost, (ii) cost incurred for contributing to meet inter-period constraints, and (iii) cost incurred for contributing to meet intra-period constraints.

It should be noted that the number of subproblems at iteration \( \nu \) is equal to the number of periods which is \( K \).

It should also be noted that the subproblems are solved independently.

The optimal solution of every subproblem is obtained from a merit order criterion. That is, plants are arranged from lower to higher production cost, forming what is called the merit order. Then, to compute expected energy productions and expected costs, they are successively loaded in the corresponding equivalent load duration curve as originally stated by Balériaux et al. [1] and Booth [2]. It should be noted that the above optimal solution procedure is independent of the number of facet constraints.

Units affected by active dispatch constraints are forced to change their location in the equivalent load duration curve and to lie in the position determined by their respective equivalent costs. The equivalent cost of one unit in one period is the cost coefficient of that unit in the objective function of the subproblem associated to that period. The equivalent cost (\( \varphi \)-value in [8]) depends on the unit running cost, the dual value of the active dispatch constraint and the linear coefficient of that unit in the dispatch constraint.

**C. Iterative Procedure**

The solution at iteration \( \nu \) of subproblem \( k \) is a “useful” solution if it has a positive reduced cost, i.e., if \( \sigma^k_{i, \nu} - \mu^k_{i, \nu} > 0 \) is positive. Any useful subproblem solution can be incorporated into the master problem as a new variable to improve the current master problem solution [11].

At iteration \( \nu \) the master problem carries out a convex combination of the “useful” subproblem solutions of the first \( \nu - 1 \) iterations with the objective to meet complicating constraints while achieving the minimum cost. The solution of the master problem provides cost signals (\( \lambda \)-s and \( \mu \)-s) to be used by the subproblems to implicitly take into account the complicating constraints.

Once every plant production cost has been updated using master problem price signals, the subproblems are solved independently. The solutions of the subproblems provide the master problem with information on the usage of the “resources” (right-hand sides) associated to the complicating constraints.

The master problem and the subproblems are solved iteratively until no useful subproblem solution is found, i.e., until no master problem variable with positive reduced cost exists.

Once the iterative procedure is completed, expected energy values of plants in every period are computed as

\[ e^k_{i, \nu} = \sum_{s \in \psi_{i, \nu}} c^k_{i, s} f^k_{i, s} + \sum_{l \in Z_{i, \nu}} c^k_{i, l} f^k_{i, l}; \]

\[ i = 1, \ldots, I + 1; \quad k = 1, \ldots, K \quad (22) \]

The computational efficiency of the above procedure relies upon the following two facts: (i) incorporating inter-period complicating constraints to the master problem makes it possible an independent solution of the subproblems, and (ii) incorporating intra-period complicating constraints to the master problem makes it possible a straightforward solution of the subproblems through the use of the Balériaux technique [1].

It should be noted that the convergence and robustness of the above procedure are guaranteed because all subproblems are bounded [11].

**D. Implementation Issues**

Four implementation issues are discussed below.

1. At any iteration, all useful subproblem solutions can be incorporated into the master problem, and not only just one. This strategy has proved to be particularly efficient because the number of master problem iterations is significantly reduced, while the additional computational burden associated to a higher number of columns is negligible.

2. A “warm” start for the master problem is possible after the first iteration, because the master problem solution of the previous iteration can be used as an initial solution for the master problem of the current iteration.

3. Although the Dantzig–Wolfe decomposition procedure always converges to the optimal solution of the original problem [11], for practical reasons, it is more appropriate to stop the procedure when the per unit difference between the upper bound and a lower bound of the objective function of the original problem is below a pre-specified threshold \( \epsilon \).

4. Not all subproblems have to be solved at each iteration. Any subproblem, whose updated unit production costs do not modify the Balériaux loading order of any previous iteration, does not have to be solved. This is so because the expected energy values associated to that subproblem do not differ from the ones of that previous iteration.
Fig. 1. Structure of master problem matrix.

Fig. 2. Convergence behavior of the Dantzig–Wolfe technique.

E. Initialization

For every period, one or several solutions of the corresponding subproblems are easily generated by using the Balériaux technique. With all these subproblem solutions a first master problem is solved. The subproblem solutions should be generated so that this first master problem is feasible. This is easily accomplished due to the structure of the multi-period probabilistic production cost problem. Then, the algorithm proceeds as previously stated.

IV. CASE STUDIES

The example analyzed in detail in [8] includes 5 thermal plants and 2 pumped storage plants. It is a single period example. The procedure developed in this paper has been applied to that example obtaining exactly the same results. The number of iterations required by the master problem was 13, and the CPU time required on a Pentium PC with 32 MB of RAM was 0.22 seconds. It should be noted that the computational efficiency of the developed procedure is more apparent when solving large-scale multi-period case studies, as the one reported below.

A comprehensive case study based on the generating system of mainland Spain is presented. Thermal plants includes 7 nuclear and 48 coal-fired units. Eight cascaded hydro plants representing the Douro river are considered. The planning horizon is one year divided in 12 periods. For each period a load duration curve is used.

The case study analyzed considers 17 dispatch constraints: 12 intra-period must-run constraints that couple together expected energy values of 5 thermal plants within each period, 4 inter-period environmental constraints that couple together expected energy values of 4 thermal units along 3 consecutive periods, and finally 1 inter-period hydro-electric constraint that couples together expected energy values of the 8 hydro units involving all periods. Must-run constraints enforce take-or-pay fuel contracts, environmental constraints enforce pollutant emission quotas limiting environmental impact, and the hydroelectric constraint couples hydro energy productions of all periods. The complete LP formulation of this case study requires $12 \times (2^{(63+1)} - 1) = 2.21 \times 10^{20}$ facet constraints.

The number of rows of the master LP problem matrix is 29 (17 dispatch constraints and 12, one per period, convex con-
Probabilistic production cost models are widely used to compute expected values of energy production and cost in a medium/long-term horizon. Probabilistic production cost models based on the equivalent load curve are computationally efficient and accurate tools.

However, modeling dispatch constraints is not possible unless a facet LP formulation is used. This paper uses a facet LP formulation to extend the traditional single-period probabilistic production cost model to a multi-period framework. Both, inter-period and intra-period dispatch constraints are considered.

An efficient and general solution technique based on the [10] Dantzig–Wolfe decomposition technique is developed to solve this multi-period problem.

Results based on a realistic and large-scale case study are reported.

V. Conclusions

Probabilistic production cost models are widely used to compute expected values of energy production and cost in a medium/long-term horizon. Probabilistic production cost models based on the equivalent load curve are computationally efficient and accurate tools.

However, modeling dispatch constraints is not possible unless a facet LP formulation is used. This paper uses a facet LP formulation to extend the traditional single-period probabilistic production cost model to a multi-period framework. Both, inter-period and intra-period dispatch constraints are considered.

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