Segmentation of vessels from mammograms using a deformable model

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Summary Vessel extraction is a fundamental step in certain medical imaging applications such as angiograms. Different methods are available to segment vessels in medical images, but they are not fully automated (initial vessel points are required) or they are very sensitive to noise in the image. Unfortunately, the presence of noise, the variability of the background, and the low and varying contrast of vessels in many imaging modalities such as mammograms, makes it quite difficult to obtain reliable fully automatic or even semi-automatic vessel detection procedures. In this paper a fully automatic algorithm for the extraction of vessels in noisy medical images is presented and validated for mammograms. The main issue in this research is the negative influence of noise on segmentation algorithms. A two-stage procedure was designed for noise reduction. First, a global approach phase including edge detection and thresholding is applied. Then, the local approach phase performs vessel segmentation using a deformable model with a new energy term that reduces the noise still remaining in the image from the first stage. Experimental results on mammograms show that this method has an excellent performance level in terms of accuracy, sensitivity, and specificity. The computation time also makes it suitable for real-time applications within a clinical environment.

1. Introduction

The processing of medical images to obtain vessel features is a prior step in the automation of diagnosis processes for a number of diseases in medicine. This problem has been dealt with by several authors who have developed different methods during the last 15 years [1—3]. This task can be very complicated to handle when the vessel structure is complex or the images available for a specific application are noisy images. In fact, the presence of noise, the variability of the background, and the low and varying contrast of vessels in most of the images from mammograms make it quite difficult to obtain reliable results with the vessel segmentation algorithm used in angiograms or other approaches to medical images. We present a new fully automated algorithm that is based on a deformable model and is suitable, both in terms of results and computation time, for the segmentation of vessels in medical noisy images such as mammograms. Deformable models have been applied successfully to medical imaging in the last decade [4—10]. Amini et al. [11] applied an iterative algorithm where, at each step, Dynamic Programming...
In Section 4, we report the results of a deformable model for mammograms is described. Our new approach based on computer vision techniques. Basically, all of them try to achieve this objective by analyzing the whole image and taking into consideration certain anatomical knowledge. Unlike other approaches, using neural networks [19] and three-dimensional vessel recognition have been reported [1, 20]. In terms of accuracy, sensitivity, and specificity these methods offer quite reasonable performance levels. The disadvantage of these methods is that the complexity of the algorithms imposes computation times much greater than those required by the best tracking method.

3. Objectives

A vessel detection algorithm is often used as part of a diagnosis process in conjunction with other algorithms to build an automatic system. Thus, the computation time of the vessel detection algorithm must be added to the computation time of all the algorithms involved in the automatic process. In our approach to the detection of microcalcifications (MC) in mammograms, the segmentation of vessels needs to be performed in order to eliminate the vascular false positives [21]. Thus, vessel detection is used in conjunction with the MC detection algorithm. Therefore, the computation time of the vessel detection algorithms is crucial to obtaining a reliable automatic system to be used in a clinical environment. In addition, an algorithm with low computational cost could be used not only in static imaging applications but also in motion.
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Imaging applications in biomedicine, such as video angiograms or cellular tracking in a sequence of microscope images [1]. In the context of applications with images having a high level of noise, none of the fully automatic methods summarized in the previous section are robust enough to handle the high level of noise present in the image or they have a computational cost which is too high to be used in a real-time application.

The objective of this study is to develop an algorithm for vessel detection in images with high levels of noise and obtain the highest possible performance both in terms of quality of results and complexity of the algorithm, while maintaining the algorithm as close as possible to real-time processing.

The algorithm is validated with vessel segmentation on mammograms from a database of 13 patients comprising a total of 52 mammograms.

4. Algorithm

The main issue to be taken into account in the development of the algorithm is that it must perform a segmentation of the vessel while keeping the loss of information as low as possible which is the aim in any noise reduction process. This algorithm uses two approaches applied in two stages: global and local approaches.

- The global approach is achieved by the use of an edge detector and a threshold that eliminates all the image points with a gray level below a certain value. This stage must be handled carefully because a bad choice of the threshold value can produce a serious loss of information leading to erroneous segmentation results. A theoretical study of different edge detectors for the selection of the best edge detector and threshold value is presented.
- The local approach is based on a snake deformable model that surrounds the contour of the vessel. Noise elimination is carried out by the introduction of a new energy term in the snake model that is aware of the noise particles crossed by the snake in the convergence process and which still remain in the image (from global noise reduction).

1 We use the term real-time processing as in [1], that is, hardware and software which is able to process at least 10 images per second, a rate that can still give a human observer the illusion of continuity of motion.

Fig. 1 shows a graph with a scheme of the algorithm. For the elimination of vascular false positives in the detection of MC the whole image does not need to be analyzed, only the region of interest (ROI) where a cluster of MCs has been found. In the first stage, the global approach, the ROI is the input of the algorithm. Next, an edge detector operator is applied to the ROI. The type of edge detector and setting parameters are extracted from the edge detector evaluation that depends on the image features of a particular application, in our case, mammogram images. As a result of the edge detection operator an image containing information about vessels is obtained. A partial thresholding is performed for noise reduction. A setting threshold value is also obtained from the edge detector evaluation. This image is introduced as input to the local approach stage, where the contour snake is initialized with a circumference. The following iterations adapt the snake to the vessel. Finally, when snake segmentation is completed, the vessel is extracted via the snake’s contour. In the following, the two stages of the algorithm are described in detail.

4.1. Global noise reduction

An important issue in this study is the correct choice of the edge detector to obtain the highest signal to noise (S/N) ratio in the image. One important fact in the development of segmentation techniques is that no general theory exists [22], and ad hoc solutions are traditionally proposed [23]. Although some initial attempts in the direction of a unified theory were reported by [24], this problem is far from being solved, and none of the developed techniques are generally applicable. Given a particular application, finding the appropriate segmentation algorithm is still a problem ([22]). Performance evaluation and the comparison of competing techniques are indispensable for a particular application. A metric for measuring the performance evaluation of edge detector techniques in very noisy images such as mammograms is developed in this work.

4.1.1. Edge detector evaluation

Images are processed by use of different edge detectors followed by thresholding to produce binary images. The adjustment of the threshold is critical for image post-processing purposes. We have developed a specific metric for performance evaluation of different edge detectors that includes thresholding. Thus, as shown in Fig. 2, our technique takes the input image (a) to be evaluated and, after applying edge detection, a set of
resulting images (d) is produced by multiple thresholding (c). Our evaluation criterion (f) compares each resulting image with the true (ideal edges) image (e) in order to establish edge detector performance. The highest performance value (h) is then assigned as the performance of the input image.

In the following, the metric developed for measuring edge detector performance is introduced.

4.1.1.1. Metric. The metric developed for measuring edge detector performance is based on four features: Location, Matching, Unmatching, and Spurious. The definitions for feature development follow.

Definition 1. Distance between a pixel and an image. Let $f$ be a binary image described by the $M \times N$ matrix whose $(m, n)$-th element is denoted by $f(m, n)$,
We define a distance between a pixel position, \((m, n)\), in an image, \(f\), by the expression
\[
d((m, n), f) = \min_{s \in A_f} \sqrt{(m - s)^2 + (n - s)^2}
\]  
(1)

Definition 2. Matched pixel at level one. Let \(f'_0\) be an image to be evaluated and \(f\) is the true image. A matching process between both images at pixel level is then carried out in order to compare them, as follows: Given a pixel \((m, n)\) in image \(f\), \((m, n) \in A_f\), we say that it is a matched pixel, at level one, if there exists a pixel \((m', n') \in A_{f'_0}\), such that it verifies:
\[
d((m, n), f'_0) = d(m', n', f)
\]  
(2a)
\[
d((m, n), f'_0) \leq \rho
\]  
(2b)
where \(\rho\) is the distance threshold. That is, a pixel \((m, n)\) belonging to the true image \(f\) matches a pixel \((m', n')\) at level one in the resulting image \(f'_0\) if \((m', n')\) is the closest pixel in the image \(f'_0\) to the pixel \((m, n)\) and the pixel \((m, n)\) is the closest pixel in the image \(f\) to the pixel \((m', n')\). Moreover, the Euclidean distance between \((m, n)\) and \((m', n')\) must be less than or equal to \(\rho\). If two pixels \((m', n')\) and \((m'', n'')\) verify (a) and (b), that is, both have the same Euclidean distance to the pixel \((m, n)\), \(d((m, n), f'_0) = d(m', n'), f) = d(m'', n''), f)\), either one is chosen in an arbitrary way.

Definition 3. Matched pixel at level \(k\). Let \(f'_k\) be the image obtained from image \(f\) by eliminating the matched pixels at level one, and \(f''_k\) the image obtained from image \(f'_k\) eliminating the matched pixels at level one. Then, \(A'_k \subseteq A_f\) and \(A''_k \subseteq A'_k\).

Similarly, given a pixel \((m, n)\) in image \(f''_{k-1}\), \((m, n) \in A''_{k-1}\), we say that it is a matched pixel at level \(k\) in image \(f'_{k-1}\) if there exists a pixel \((m', n') \in A''_{k-1}\), such that it verifies:
\[
d((m, n), f''_{k-1}) = d(m', n', f'_{k-1})
\]  
(3a)
\[
d((m, n), f''_{k-1}) \leq \rho
\]  
(3b)
where \(f''_{k-1}\) and \(f'_{k-1}\) are the images obtained from \(f''_{k-2}\) and \(f'_{k-2}\), respectively, eliminating the matched pixels at level \(k-1\). Then \(A''_{k-2} \subseteq A''_{k-1}\) and \(A'_{k-2} \subseteq A'_{k-1}\).

The pixels matched at level one are those that verifies Eqs. (2a) and (2b) directly, that is the set of pixels closer to each other in both images. Subsequent levels match remaining pixels between obtained and ground truth images.
Definition 4. Matched images \( f^* \) and \( f^*_o \) of \( f \) and \( f_o \). Let \( r \) be the integer such that,
\[
A^r_f = A^{r-1}_f \subset \ldots \subset A^1_f \subset A^r_f
\]
and
\[
A^r_o = A^{r-1}_o \subset \ldots \subset A^1_o \subset A^r_o
\]
that is, \( r \) is the first level where there are no new matched pixels between both images and, consequently the matched process is finished. Therefore, we define \( f^* \) and \( f^*_o \), matched images of \( f \) and \( f_o \), respectively, as:
\[
f^*(m, n) = \begin{cases} 
1 & \text{if } (m, n) \in (A^r_f - A^r_o) \\
0 & \text{otherwise}
\end{cases}
\]
and
\[
f^*_o(m, n) = \begin{cases} 
1 & \text{if } (m, n) \in (A^r_o - A^r_f) \\
0 & \text{otherwise}
\end{cases}
\]
That is, \( f^* \) and \( f^*_o \) contain the matched pixels for \( f \) and \( f_o \), respectively, at every level. This matching process has the following properties:
1. Single matching;
2. closest matching pixel guaranteed.

The matching process has been used for the evaluation of segmentation in previous works [25], but either no closest pixel matching was guaranteed or multiple matching was allowed. Fig. 3 show a portion of true and resulting edges during the matching process. Pixels \( a, b, c, d \), and \( e \) belong to the resulting edge and pixels \( f, g, h, i, j, \) and \( k \) belong to the true image. If multiple matching is allowed then both pixels \( h \) and \( i \) are matched with \( c \). This is not a correct matching because \( f \) should be matched with \( c \) and pixel \( h \) classified as spurious. If multiple matching is not allowed then the result depends on the direction of execution (left to right or, right to left). Considering the left-to-right direction then

\( h \) is matched with \( c \), \( l \) is matched with \( d \), and \( j \) is matched with \( e \). Considering the right-to-left direction then the matching will be \((k, e), (j, d), (l, c), (h, b) \) and \((g, a)\). Both of these are wrong matching. Our method performs the correct matching which is \((f, a), (g, b), (l, c), (j, d), (k, e)\) and \(h \) is classified as spurious, and this result is independent of the direction. The correct matching is reached due to the two properties of the matching expressed in Eqs. (3a), (3b) and (4).

In the following, the features of the metric are described.

4.1.1.2. Distortion (D). This feature measures how close the detected edge is to the ideal edge present in the true image \( f \). The expression for \( D \) is
\[
D(f_o, f, \rho) = \frac{\sum_{(m, n) \in A_f} D((m, n), f^*_o)}{|A_f|} \rho
\]
where
\[
D((m, n), f^*_o) = \sqrt{(m - m^*)^2 + (n - n^*)^2}
\]
and \((m^*, n^*)\) is the matched pixel in image \( f^*_o \) of pixel \((m, n)\).

This measure, \( D \), has a range from 0 to 1. If the localization of the edge is perfect, \( D = 0 \). The worst localization value will be \( D = 1 \). Notice that this parameter does not consider the pixels that do not have a match for the resulting image, that is, the part of the edge that was not detected.

4.1.1.3. Matching (M). This feature indicates the percentage of the resulting edge successfully detected:
\[
M(f_o, f, \rho) = \frac{|A^r_f|}{|A_f|}
\]
\( M \) is normalized by the number of pixels in the true image \( f \) and \( M = 1 \) if all the pixels of the true image have been matched.

4.1.1.4. Spurious (S). To find possible multiple responses, non-matching pixels must be calculated. This is carried out by the following feature:
\[
S(f_o, f, \rho) = \frac{|A^r_o| - |A^r_f|}{|A^r_o|}
\]
\( S \) also ranges from 0 to 1. Two different reasons can cause a resulting pixel not to be matched: a pixel closer to the ideal edge was already matched—and presumably this pixel will be a multiple response to the edge—or this pixel is too far from the edge (distance > \( \rho \)) to be matched, and it is part of an edge (or noise) not related to the ideal edge.
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4.1.1.5. Performance (P). A new term for performance evaluation is introduced from the above metric features based on the defined matching process (previous works [25] lack a precise matching function). By combining the three defined features \( M, S, \) and \( D \) in a single expression, the performance value can be obtained by expression,

\[
P = \begin{cases} 
\frac{M}{S + T} & M \neq 0 \\
0 & M = 0
\end{cases}
\] (11)

When perfect detection is obtained, \( M = 1, S = 0, \) and \( D = 0, \) then \( P = 1. \) An example showing the input and resulting and ideal images can be seen in Fig. 4. The ideal image for each input image was extracted by a radiologist. Matched, unmatched, and spurious pixels are also shown.

After the performance is obtained for a particular application and for all edge detector techniques, then the best one is chosen.

4.1.2. Thresholding

The threshold operation is achieved with the aim of reducing noise in the image. Multiple thresholding is applied to obtain binary images, which are suitable for evaluation. In multiple thresholding, given an image and an edge operator, the threshold value that supplies the best binary image (highest performance) is called the optimum threshold. The average optimum threshold is calculated for every edge detector technique. This is used to eliminate image pixels with a gray level below the threshold value, while maintaining the remaining image pixels unchanged.

4.2. Local noise reduction

A deformable model-based segmentation scheme can overcome many of the limitations of traditional segmentation techniques. In continuous geometric models an object boundary is considered as a whole and is used as a priori knowledge of the object’s shape to constrain the segmentation problem. The inherent continuity and smoothness of the models can compensate for gaps and other irregularities in object boundaries which are very common in vessels in mammograms. Furthermore, the parametric representations of the models provide a compact and analytical description of the object’s shape.

In the initial approach of Kass et al. [13], the contour is represented by a vector, \( v(s) = (x(s), y(s)) \), where \( s \) is the arc length. The energy along the snake is calculated as follows:

\[
E_{\text{snake}} = \int_0^1 \left[ E_{\text{int}}(v(s)) + E_{\text{image}}(v(s)) + E_{\text{ext}}(v(s)) \right] ds (12)
\]

where \( E_{\text{int}} \) represents the internal energy of the contour due to bending or discontinuities, \( E_{\text{image}} \) is related to image forces, and \( E_{\text{ext}} \) indicates the external constraints. The image forces can be due to various events such as lines, edges, or terminations [13]. The internal spline energy is represented by a first-order term which generates larger values in curve gaps. The second-order continuity term takes larger values where the curve bends sharply.

\[
E_{\text{int}}(v(s)) = \left( \alpha(s) \left| \frac{dv(s)}{ds} \right| + \beta(s) \left| \frac{d^2v(s)}{ds^2} \right| \right) / 2 (13)
\]
The $\alpha$ and $\beta$ coefficients are used to determine the strength with which the contour is allowed to stretch or bend at a point.

The adaptations applied to the basic snake model to adapt it to mammogram images focus on solving the two problems (bunch-up and noise sensitivity) previously referred to in Section 1.

4.2.1. Solving the ‘bunch-up’ problem

During the minimization process a tendency of points to ‘bunch up’ on dense portions of a contour is sometimes present. This effect has a negative influence on the snake for vessel contour representation. Williams and Shah [12] reformulated the first-order continuity term that causes the points to be evenly spaced on the contour, thus eliminating the tendency of the contour to shrink. This solution forces the initial contour to be close to the solution (object contour). Choi et al. [26] tried to solve this problem by the insertion and deletion of snake points keeping the distance between each point close to a constant, but the ‘bunch-up’ problem may still be present and all the snake points can be deleted. In our deformable model this new condition are presented.

Let, $(P_i, P_{i+1}, \ldots P_n)$ be a discrete and closed contour of $n$ points, with $P_0 = P_n$. The average distance $d_m$ is defined as:

$$d_m = \frac{\sum_{t=1}^{n-1} |P_i - P_{i+1}|}{n} \tag{14}$$

If $P_i(t)$ represents the position of a point $P_i$ at iteration $t$, we can define $P_i(t+1)$ as follows:

$$P_i(t + 1) = B_i[G(P_i(t)), P_{i-1}(t)] \tag{15}$$

where $G(P_i)$ is the point with the highest value for the gradient of the energy function, within a circle of radius $r$ centered at point $P_i$. In addition, the expression for the $B_i$ function is

$$B_i(P_i, P_{i-1}) = \begin{cases} P_i & \text{if } d_m > |P_i - P_{i-1}| \\ \frac{d_m P_i}{|P_i - P_{i-1}|} + P_{i-1} \left( 1 - \frac{d_m}{|P_i - P_{i-1}|} \right) & \text{if } |P_i - P_{i-1}| - d_m \leq \lambda d_m \\ |P_i - P_{i-1}| - d_m > \lambda d_m \end{cases} \tag{16}$$

that is, if the distance between the new location $P_i$ and the adjacent point $P_{i-1}$ is below the threshold $\lambda d_m$, then $P_i$ is fixed as the new location of point $P_{i-1}$ at the next iteration. In other cases, $P_i$ is set to a vector of magnitude $d_m$ having the same direction as $(P_i - P_{i-1})$.

4.2.2. Noise problem

Noise detection can be carried out by the incorporation of local pixel information. For this purpose, we define a new energy term, $E_{noise}$.

Let us consider the binary edge image, $e(m, n)$, obtained from gradient image $E_{image}$ with threshold $t$. Let $C_i(m, n)$ be the set of pixels belonging to the boundary of a neighborhood of $(m, n), N(m, n)$, with radius $r$. That is,$$
C_i(m, n) = \{(t, s) \in Z_i \times Z_i : d((t, s), (m, n)) = r\} \tag{17}
$$

where $d$ is a distance function such as a rectilinear distance or Euclidean distance.

If a radius $r$ is fixed, we say that a pixel, $(m, n)$, from the image, where $(m, n)$ belongs to the active contour, is a granular noise pixel if

$$\sum_{(i, j) \in C(m, n)} e(i, j) = 0 \tag{18}$$

that is, it is a contour pixel with no active points to a distance $r$ in the binary edge $e$ image. The $(m, n)$ pixel is considered as an ambiguous pixel if

$$\begin{align*}
&\|P'_i - P_{i-1}\| - d_m \leq \lambda d_m \\
&\|P'_i - P_{i-1}\| - d_m > \lambda d_m
\end{align*} \tag{16}$$

\begin{enumerate}
  \item $\sum_{(i, j) \in C(m, n)} e(i, j) \neq 0$,
  \item $C_i(m, n)$ is a connected set,
  \item $\text{curv}(m, n) = c$, where $\text{curv} (\mathbf{v}) = \frac{d^2}{dx^2} \approx |v_x|^2 - v_{x - 1}^2 = (x_i - x_{i - 1})^2 + (y_i - y_{i - 1})^2$ and $c$ is a fixed parameter,
\end{enumerate}
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Fig. 6 Local noise reduction. A circumference of radius \( r \) is analyzed to classify each pixel as granular noise, an ambiguous pixel or a local edge point. \((m_1, n_1)\) refers to a noise particle.

where (a) means that some points at a distance \( r \) from \((m, n)\) are active (set to 1), (b) means that all these points are contiguous (connected), and (c) means that the curvature contour at point \((m, n)\) is above a threshold \( c \) (when the contour is captured by a noise particle at a point, that point usually has a high curvature value as shown in Fig. 6). In other cases, the pixel \((m, n)\) is considered as an edge pixel.

Now, we propose noise energy at pixel \((m, n)\) by expression

\[
E_{\text{noise}}(m, n) = \begin{cases} 
-E_{\text{image}}(m, n) & \text{if } (m, n) \text{ is a granular pixel} \\
-\frac{1}{2}E_{\text{image}}(m, n) & \text{if } (m, n) \text{ is an ambiguous pixel} \\
0 & \text{if } (m, n) \text{ is a local edge} 
\end{cases}
\] (19)

Note that if \((m, n)\) is a granular noise pixel then the noise energy neutralizes the image energy at \((m, n)\). Thus, the snake is not attracted by the pixel. When \((m, n)\) is a local edge then the noise energy is zero and if it is an ambiguous pixel then the image energy is decreased by half.

The definition of granular noise is based on these noise features, but it can be extended to different kinds of noise in a particular application. In this work, we apply snake techniques to images from mammograms where the majority of the noise is produced by small artifacts. Thus, two assumptions have been made: noise is spread over small regions in the image, and these regions are too far from the object to be segmented or are other noisy regions.

Fig. 6 illustrates different situations for the consideration of granular noise. The new \( E_{\text{noise}} \) value is included as a term in the general expression of the snake energy:

\[
E_{\text{snake}} = \int \left[ \alpha(s)E_{\text{cont}} + \beta(s)E_{\text{curv}} + \gamma(s)E_{\text{image}} + \delta(s)E_{\text{noise}} \right] ds
\] (20)

This new term eliminates the attraction that small noise regions exert on the snake and thus prevent it from being trapped.

An example showing the steps of the algorithm can be observed in Fig. 7. A circumference has been drawn around the image as an initial contour.

5. Experiment results
In order to show the robustness and sensitivity of the vessel segmentation algorithm described in the previous section, we have used a database of 13 cases, with four mammograms per case (52 mammograms in all) containing 115 vascular false positives. The mammograms were digitized at 100 μm. A ROI was extracted from each vascular false positive to obtain a set of 115 images, 128 × 128 pixels in size.

Fourteen edge detection methods were evaluated with the developed metric, i.e. first-order derivative methods (Canny, Difference, Separate Difference, Frei-Chen, Prewitt, Robert, and Sobel), second-order derivative (Laplacian Four-Neighbor, Eight-Neighbor, and Separable Eight-Neighbor), and directional gradient (Base, Kirsh, Qiu, and Schalkoff). In the global approach, the 115 images were segmented and evaluated for performance calculation with every edge detection algorithm and the average performance for all images was calculated. Fig. 8 shows the result of this evaluation. The best performance obtained was for the Canny edge detector. In terms of best results, average performance is not a very precise measure of good segmentation because mammogram images have
different contrasts and the result of a detector technique can be very different between images. We have taken into account deviation of performance since it is indicative of how sensitive a detector is to different image characteristics. Fig. 9 shows the standard deviation (S.D.) for all edge detector techniques. The best method (lowest S.D.) was the Canny operator which means it is the detector
least sensitive to different image characteristics. Globally, the results indicate that the Canny edge detector is the best method to be used in the edge detection of vessels in mammograms. The average optimum threshold $t$ for the 115 images was 137 over a maximum of 255 gray levels and a S.D. of 0.046.

To validate the metric as a performance measure of the edge detectors and threshold, a theoretical study was also carried out using synthetic images. These images were generated as the sum of a contour vessel extracted manually from a real mammogram (true image) and additive noise (see Fig. 10). The noise present in a mammogram can be modeled as the result of two components: high frequency noise and low frequency noise. The high frequency is originated by MC, macrocalcifications, tissue structures and artifacts from the acquisition machine (digital or analog mammogram machine). The low frequency noise is originated by fat appearing in the breast and it has a smoothing effect over the mammogram image. Often, this smoothing effect is so aggressive that it causes part of the vessel be hidden. In our synthetic images, the high frequency noise component of the mammogram is generated as a Perlin noise function \cite{27,28} that simulates the noisy background of the mammogram images. Perlin noise functions have been widely used as a texture generator (for simulation of textures from nature) in movies and video games. A Perlin noise

![Fig. 9](image_url)  
Fig. 9  S.D. of edge detector evaluation.

![Fig. 10](image_url)  
Fig. 10  Synthetic image (right) obtained from a Perlin noise texture (left) that stimulate the noisy background (high frequency noise) of the mammogram plus a vessel (center) manually extracted from a real mammogram (bottom center) and smoothed (low frequency noise, upper center).
function is a coherent noise obtained from the addition of a succession of non-coherent noise functions. Some synthetic images are shown in Fig. 11. In the second stage, the local approach, the snake model was applied to the images resulting from the global approach. The initial contour for the snake was set as a circumference that circumscribes the image or ROI from the mammogram. The number of iterations needed for the snake to converge ranged from 77 to 149. The average number of iterations was 121. The value of the curvature \( \text{Curv}(m, n) \) was normalized and the threshold of curvature \( c \) was empirically set. This parameter is crucial for...
a correct pixel classification as granular noise, local edge or an ambiguous pixel. When the active contour is trapped in a point by a particle noise, a high curvature is produced on that point (Fig. 6). If the threshold $c$ is too high then a possibly ambiguous pixel is always considered as local edge of the vessel affecting to the performance of snake segmentation process. Nevertheless, if the threshold $c$ is too low then every discontinuity on the edge can be considered as an ambiguous pixel and then the vessel cannot be detected. In Fig. 6, the curvature value of three contour points is shown. The curvature at the contour point trapped by the particle noise $(m_1, n_1)$ is 0.74 and the curvature at the two contour points on the edge $(m_2, n_2)$ $(m_3, n_3)$ are 0.36 and 0.57, respectively. Then, it was checked empirically that a value of 0.63 was adequate for the threshold curvature $c$.

In some cases where the vessel has a convex section (Fig. 12), the basic snake model is unable to

![Figure 14](image-url) Results obtained in some images and the fitting coefficient for each one.
enter this convex section. This problem was solved by implementing the solution proposed by [29].

To measure the quality of the results, the image obtained was compared with the true image through a fitting criteria ($F$), where $F=1-D$ and $D$ is the distortion parameter (see Section 4.1.1.2) (Fig. 13).

Over the 115 tested images, it was checked that a value greater or equal to 0.70 is a good threshold to consider a correct snake fitting. The vessels were correctly segmented ($F ≥ 0.70$) in 94 images (82%) (for example, see Fig. 14, a, b, c, d, g, i, j, m, n, o, q, r, s and t) whereas in the other 21 images (for example Fig. 14, e, f, h, k, l, p) the snake was not able to segment correctly the vessel because it was trapped by the noise or a fragment of the vessel was missing. However, just in six of these 21 images (5%) (for example Fig. 14i) the results cause the vascular false positive is not detected. Some images have a not detected fragment (Fig. 14h and p) but the rest of the vessel is correctly detected. However, vessel extraction in these images is difficult even for human eye.

Fast results are also very important when using this technique for the elimination of vascular false positives in the context of a clinical environment. This algorithm was implemented in a standard PC $1 
\text{GHz Pentium III,} 128\text{MB RAM memory, running redhat language.}$

6. Discussion and conclusion

In this paper we have presented an algorithm for the segmentation of vessels in mammograms which is very useful for the elimination of vascular false positives during detection of MC in mammograms. The main problem is the high level of noise present in mammograms. Two approaches (stages) have been used to deal with this problem. First, a global approach where a theoretical analysis of edges detectors is carried out in order to select the optimum edge detector and threshold value. This is applied to the whole image to improve the S/N ratio thus avoiding significant signal loss. Second, a local approach, where local noise—represented as particle noise, remaining in the image from the global noise reduction—is removed by a segmentation process based on a snake with a new noise energy term which extracts the vessel contour.

The results obtained—both in terms of performance and computation time—indicate that this algorithm is suitable for being used in a clinical environment. The 82% of the vascular false positives in the detection of the MC in mamograms are eliminated taking just an extra second in time for mammogram in this process, where the vascular false positive fraction represents about 10–15% of the total false positive [21]. We are currently working for adapting this technique to other medical images such as angiograms.

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