Outage probability with MRC in presence of multiple interferers under Rayleigh fading channels

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Closed-form analytical expressions are derived for the outage probability of predetection maximal ratio combiner (MRC) diversity reception with \( M \) branches in Rayleigh fading channels. This formulation is obtained in the presence of white Gaussian noise and \( Q \) interferers, without restrictions regarding the power, number or distribution parameters of interferers.

Introduction: Antenna diversity can improve the capacity of digital cellular radio systems combating multipath fading and reducing the effect of co-channel interference (CCI). Fig. 1 shows a schematic presentation of the diversity structure. The freedom to select the weight allows different techniques to exploit diversity. One technique is the maximal ratio combiner (MRC), which offers the best performance when only additive white Gaussian noise (AWGN) is present. Even if CCI is not neglected, MRC is a sub-optimal technique, but its easiness for implementing and computing situates it as the preferred and a very useful technique. The performance of MRC systems has been analysed under several conditions. The solution for the outage probability (OTP) has been studied with Rayleigh fading channels by Shah and Haimovich [1], without noise and with equal power for interferers, using hypergeometric functions. Also the Rice probability distribution function (pdf) is analysed for the signal of interest (SOI). Cui and Sheikh [2] obtained a close-form solution with noise, but limited by the equal-power interferers, which is a very restrictive situation. Aalo and Chayawan [3] avoid such limitation and a general analytical solution is shown, but the powers of interferers have to be all different or all equal. No correlation between branches of the array is assumed in all of these cases, which is rather suitable when distance between branches is smaller than one-half carrier wavelength. However, Annamalai et al. [4] show a very deep analysis of OTP with a lot of possible pdf for signals and interferers, but in single antenna systems.

Fig. 1 M-branch adaptive antenna with \( Q + 1 \) users

The present work contributes to obtaining closed-form analytical and easily computable results, applicable in a vast range of real cases for Rayleigh fading channels. Most of the restrictions present in previous work have been removed.

System model: In the following, capital bold letters are reserved for matrices, small bold letters are for column vectors, and * denote complex conjugate and Hermitian transpose (transpose and complex conjugate), respectively. As can be seen from Fig. 1, the output of an \( M \)-element antenna array operating in the presence of \( Q \) co-channel interferers is the sum of the signal received by each antenna weighted according to which technique is used:

\[
y(t) = \mathbf{w}^H \cdot \mathbf{x}(t) = \sum_{q=0}^{Q} w_q^H \cdot x_q(t) = \sum_{q=0}^{Q} y_q(t)
\]  

(1)

where the vectors have as many components as branches in the array (\( M \)), and \( q = 0 \) represents the SOI. We assume flat fading. Therefore, signal bandwidth is smaller than channel bandwidth. The channels are assumed to be well known. We assume that AWGN is present (vector \( n \), not neglected, and with the same behaviour for the entire array, with mean power \( \sigma^2 \)). Therefore, the mean power received from a mobile \( q \) over each branch is assumed to be the same and fixed during the considered period, and denoted by \( P_q \). The input vector in baseband to an \( M \)-element antenna array operating in the presence of \( Q \) co-channel interferers may be written as

\[
x(t) = h_0 s_0(t) + \sum_{q=1}^{Q} h_q s_q(t) + n(t)
\]  

(2)

where the subscript ‘0’ corresponds to the SOI, \( s_q(t) \) is the transmitted signal in baseband (binary) by the mobile \( q \). It is unit power normalised (\( E[|s_q(t)|^2] = 1 \)), \( n \) is the noise vector and \( h_q \) the propagation vectors from \( q \)th transmitter. According to the assumptions, their components \( h_{0q} \) are \( \delta \) dimensional pdfs \( p(h_{0q}) \) where \( \delta \) is an RV following a Rayleigh [2] pdf. The phase \( \theta_0 \) is an RV uniformly distributed over \([0, 2\pi]\). It is well known that the optimum weight vector for MRC is \( \mathbf{w}_{opt} = h_0 \). When CCI is present, MRC technique leads to:

\[
\begin{align*}
\mathbf{x}(t) &= h_0 s_0(t) + \sum_{q=1}^{Q} h_q s_q(t) + n(t) \\
&= \left( \sum_{q=1}^{Q} h_q s_q(t) + n(t) \right) + h_0 s_0(t)
\end{align*}
\]  

(2)

Outage probability: For a given power protection ratio \( \gamma \), the outage probability is defined as:

\[
\text{OTP} = \Pr \left( \frac{x_0}{\sum_{q=1}^{Q} z_q + \sigma^2} \leq \frac{\gamma}{\gamma} \right)
\]  

(8)

Then

\[
\text{OTP} = 1 - \Pr \left( \frac{x_0}{\gamma} \geq \frac{\sum_{q=1}^{Q} z_q + \sigma^2}{\gamma} \right) = 1 - P
\]  

(9)

where, if \( z = \sum_{q=1}^{Q} z_q \),

\[
P = \int_{0}^{\infty} f_z(z) \, dz
\]  

(10)

From (6) and because \( M \) is integer, the dimension of the array, it is easy to obtain that

\[
\int_{0}^{\infty} f_z(z) \, dz = e^{-(\gamma+1)/\gamma} M^{-1} \frac{1}{\gamma^M} \left( \frac{1}{P_0} \right) \sigma^2^M (z + \sigma^2)^M
\]  

(11)

where \( P_0 = P_0/\gamma \). Assuming independence between interferers and if we consider the multinomial theorem [5]...
\[
\left(\sigma^2 + \sum_{q=1}^{Q} z_q\right)^k = \sum_{k_1, \ldots, k_Q} \frac{k!}{k_1!k_2! \cdots k_Q!} \sigma^{2k_1} z_1^{k_1} \cdots z_Q^{k_Q}
\]
for all combinations \( \sum_{q=1}^{Q} k_q = k \), \( k_q \in \mathbb{N} \) \( k_1, \ldots, k_Q \) (12)
and (7), we obtain after some manipulation
\[
P = e^{-\sigma^2/P_0} \frac{M-1}{\sum_{k=0}^{M-1} (P_0/P_q)^k} \sum_{k=0}^{\infty} \frac{\sigma^{2k}}{k!} \frac{1}{P_q} \prod_{q=1}^{Q} \frac{1}{P_q} \int_0^{\infty} e^{-z} [1/(P_0/P_q)^{k_1} + (1/P_q)]^{k_1} dz_q
\]
(13)
But the solution of the last integral of (13) is \( \{[(1/P_0) + (1/P_q)]^{-k_1} \} \). Therefore
\[
OTP = 1 - e^{-\sigma^2/P_0} \frac{M-1}{\sum_{k=0}^{M-1} (P_0/P_q)^k} \sum_{k=0}^{\infty} \frac{\sigma^{2k}}{k!} \frac{1}{P_q} \prod_{q=1}^{Q} \frac{1}{P_q} \left[1 + \frac{1}{P_q} \right]^{-k_1-1}
\]
(14)

Fig. 2 shows some results from (14). It represents the OTP related to the global (long-term) expected value of SINR without diversity, for several array sizes.

Conclusions: We have studied the outage probability of digital cellular mobile radio communications systems in the presence of AWGN and CCI, with a reception diversity system (M-antennas) using the MRC technique. Rayleigh fading channels, as the most usual, have been considered. Closed-form easily computable expressions were derived with no restriction regarding the number or distribution parameters of interferers. The effect of diversity over the OTP is shown.

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